

A Viable Dark Fluid Model

Esraa Elkhateeb

Physics Department, Faculty of Science, Ain Shams University., Abbassia 11566, Cairo, Egypt

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Abstract

We Consider a cosmological model based on a generalization of the equation of state proposed by Nojiri and Odintsov [46] and Štefančić [47], [48]. We argue that this model works as a dark fluid model which can interpolate between dust equation of state and the dark energy equation of state. We show how the asymptotic behavior of the equation of state constrained the parameters of the model. The causality condition for the model is also studied to constrain the parameters and the fixed points are tested to determine different solution classes. Observations of Hubble diagram of SNe Ia supernovae are used to further constrain the model. We present an exact solution of the model and calculate the luminosity distance and the energy density evolution. We also calculate the deceleration parameter to test the state of the universe expansion.

1 Introduction

Recent cosmological observations favoring the scenario of spatially flat and accelerated universe. Observations come from Hubble diagram of SNe Ia supernovae are best fitted by spatially flat accelerated cosmological models [1]-[4]. Besides, other observations from Cosmic Microwave Background Radiation (CMBR) and Baryon Acoustic Oscillation (BAO) are also strong evidences for this scenario [5]-[11]. This currently observed speeding up expansion of the universe leads to the fact that there is a repulsive force acting in the universe space-time.

According to our current understanding of the universe history, the radiation era was followed by a matter dominated era during which most of the universe structures like stars, galaxies and galaxy clusters we now observe was formed by gravitational instabilities. Through this phase of the universe evolution, a particular form of gravitating but non-baryonic form of matter is formed. This sort of matter which is non-relativistic does not interact with radiation, and so is called Dark Matter (DM). Late in this matter dominated phase, a new but very peculiar form of invisible and non-gravitating "matter" started to dominate. This sort of "matter" does not interact with baryons, radiations, or any other sort of visible matter. It is now known as Dark Energy (DE).

Observational evidences show that, see for ex. [1] and [12]-[14], our universe at present contains approximately 4% only of radiation and baryonic matter that is well known by the standard model of particle physics and can be detected in laboratory and so is considered visible. While about 26% is the non-baryonic dark matter. The rest of our universe content, which is about 70% is the exotic component known as dark energy. As a result, this dominated dark energy component antigravitates leading to the present observed universe expansion.

Although dark energy is an acceptable and effective description for such large scale homogeneous and isotropic accelerated universe, identifying its origin and nature is one of the most debated questions in physics and cosmology. Large variety of models are proposed to describe the nature of such dark energy [15],[16]. As the energy density due to DE remains unchanged throughout the universe space-time, the simplest choice was to revive the idea of Einstein about the cosmological constant Λ [17],[18]. This together with Cold Dark Matter (CDM) provides the standard model for cosmology Λ CDM [18]-[21], which appears to be in a very good agreement with observational data. In Einstein's field equation, this term has the concept of intrinsic energy density of vacuum [22]. However, this arises an important question, why the observed value of vacuum energy density is very far below that is predicted from particle physics?. This is the famous fine tuning problem. Another important question is why the energy densities of the vacuum and the matter are of the same order of magnitude today?, the famous coincidence problem.

Dynamical models are other alternative to the cosmological constant. Some of these are the minimal coupled scalar fields or quintessence [23]-[26], k-essence [27]-[30], Chaplygin gas [31]-[33], all have an equation of state parameter $\omega \geq -1$. These models adopt the idea of the possibility of unification of the two dark sectors, dark energy and dark matter. Models with $\omega < -1$ known as phantom energy models [34], [35] consider a growing vacuum energy with expansion. A more general class of models considers a parameter varies with the scale factor, or the red shift [17], [20], [21], [36], [37]. There are also some observational analysis which consider the variation of ω with the red shift and with time [38], [39]. Modification of the governing equation, or the so called modified gravity theory [40], [41], is also one way of thinking.

The idea of unification of DE and DM is a promising one and Chaplygin gas model is one of the most acclaimed models representing it. This model was proposed by Kamenshchik et. al. [42] as early as 2001 where they assumed that the universe, within the framework of standard cosmology, is filled with an exotic fluid obeys the Chaplygin gas equation of state (EoS)

$$p = -A/\rho^\alpha \quad (1.1)$$

where A is a positive constant and $\alpha = 1$. This model has the advantage of smooth transition between different phases of the universe. Many authors then extended this model using the idea of generalizing or modifying the EoS of the background fluid in order to improve the behavior of the fluid throughout the universe evolution. In 2002, Bento et. al. [43] extended the model to a

more general form known as the generalized Chaplygin gas (GCG) model where the α constant is considered in the range $0 < \alpha \leq 1$. Other various models based on Chaplygin gas were also proposed such as the modified Chaplygin gas model introduced in 2002 by Benaoum [44] where he could interpolate between standard fluids at high energy densities and Chaplygin gas fluids at low energy densities using the EoS

$$p = A\rho - \frac{B}{\rho^\alpha} \quad (1.2)$$

where $\alpha \geq 1$ and A and B are positive constants, and the hybrid Chaplygin gas model introduced by Bilić et. al. in 2005 [45] and leads to transient acceleration.

In this work, we have adopted a class of dynamical models in which the EoS has the advantage of interpolation between two different Chaplygin Gas models. Our fluid is a barotropic one with an EoS can be considered as a correction to the vacuum EoS by a power law. This power law has the advantage of interpolation between the two equations of state of the DE and DM which might describe some sort of smooth phase transition. Another advantage of this model is that it is not only interpolating between dust and DE in early and late times, but also it has a more general EoS for DE that enables the cosmological constant as a special case. This has the more advantage of a general asymptotic solution than Chaplygin gas which goes to the cosmological constant. We present an exact solution of the equations which enables more accurate results for the calculations of the luminosity distance, while observations of Hubble diagram of SNe Ia supernovae are used to further constrain the model parameters. We study the energy density and the equation of state parameter evolutions. The analysis of the EoS shows that the dynamical properties of both dark sectors can be described through this model which lets our choice seems natural because of the smooth transition. We also calculate the deceleration parameter to examine the expansion of the universe due to the model.

The rest of the article is organized as follows; in the following section we have formulated the model. In section 3 the mathematical treatment of the model is made. Section 4 treats the first case of study of our model where the causality constraints are examined and the dynamics of the universe is studied where the equation of the luminosity distance as a function of red shift is solved and parameters are constrained through Hubble diagram observations, the energy density and the deceleration parameter evolutions due to the model are examined. Section 5 treats the second case of study of our model. Section 6 concludes our work.

2 Building the Model

We are working through the standard FRW cosmology where the metric in the spatially flat geometry is

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2) \quad (2.1)$$

where we consider units with $c = 1$. Using the energy momentum tensor given by

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu} \quad (2.2)$$

where in comoving coordinates $U_\mu = \delta^0_\mu$, the Einstein's equation will lead to the Friedman equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \quad (2.3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad (2.4)$$

while the conservation equation $T_\nu{}^\mu{}_{;\mu} = 0$ gives

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + P) \quad (2.5)$$

Now we have to consider an EoS to close the system. It is a fact that dark energy dominates the universe today and an adequate EoS for this component is $p = \omega\rho$. Observations indicate that the EoS parameter ω is close to -1 and people are now targeting to confirm whether ω is constant or varies as the universe expands. Owing to this fact, Nojiri and Odintsov [46] considered a general EoS of the form

$$p = -\rho - f(\rho) \quad (2.6)$$

where $f(\rho)$ is an arbitrary function may be considered as a correction to the standard DE equation of state. Štefančič [47], [48] studied in details the case of $f(\rho) \propto \rho^\alpha$ with a constant $\alpha \neq 1$, and Nojiri et. al. [49] studied the future singularity associated with this model.

In this work we adopted a barotropic fluid with an EoS which is also a correction to the standard dark energy EoS. The fluid pressure has a general power law form for the density dependence

$$P = -\rho + \frac{\gamma\rho^n}{1 + \delta\rho^m} \quad (2.7)$$

Where γ , δ , n , and m are constants which are considered as free parameters. This enables interpolation between different powers for the density, so that the phase transitions during the universe evolution are described smoothly. The EoS parameter is given by

$$\omega = -1 + \frac{\gamma\rho^{n-1}}{1 + \delta\rho^m} \quad (2.8)$$

3 The model

Let's now study the evolution of the physical quantities of the universe due to our model. Considering units with $8\pi G = 1$, eqns (2.3)-(2.5) reduce to

$$H^2 = \frac{1}{3}\rho \quad (3.1)$$

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{1}{6}(\rho + 3P) \quad (3.2)$$

$$\dot{H} = -\frac{1}{2}(\rho + P) \quad (3.3)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Combining eq(2.7) to eq(3.3) we get

$$\dot{H} = -\frac{\alpha H^r}{2(1 + \beta H^s)} \quad (3.4)$$

where again α , β , r , and s are constants related to those of (2.7) through the relations

$$r = 2n; \quad s = 2m; \quad \alpha = 3^n \gamma; \quad \text{and} \quad \beta = 3^m \delta \quad (3.5)$$

The solution of equation(3.4) indicates the evolution of the Hubble parameter with time. This equation is integrated to give

$$\frac{\alpha}{2}(t_0 - t) = \frac{1}{1-r} (H_0^{1-r} - H^{1-r}) + \frac{\beta}{s-r+1} (H_0^{s-r+1} - H^{s-r+1}) \quad (3.6)$$

However, our model is not describing a pure dark energy, in which case the values of the parameters are totally free and are not constrained. Instead, our model able to follow the smooth phase transition which takes place through the universe evolution. This restricts the parameter values to obey the limits satisfied by observations so that for early time we must have the equation of state of the perfect fluid.

Accordingly, we first have to study the asymptotic behavior of \dot{H} . From one side, this indicates to what extent our model agrees the real evolution of the universe, and from the other side may also fixes the limits of some of our model parameters. Table 1 shows this behavior for the different ranges of the parameters r and s .

Table 1: Asymptotic behavior of the equation of state			
Case	ranges of r and s	large H	small H
<i>I</i>	$r > 0$ & $s > 0$	$\dot{H} \rightarrow \frac{-\alpha}{2\beta} H^{r-s}$	$\dot{H} \rightarrow \frac{-\alpha}{2} H^r$
<i>II</i>	$r > 0$ & $s < 0$	$\dot{H} \rightarrow \frac{-\alpha}{2} H^r$	$\dot{H} \rightarrow \frac{-\alpha}{2\beta} H^{r-s}$
<i>III</i>	$r < 0$ & $s > 0$	$\dot{H} \rightarrow \frac{-\alpha}{2\beta} H^{r-s}$	$\dot{H} \rightarrow \frac{-\alpha}{2} H^r$
<i>IV</i>	$r < 0$ & $s < 0$	$\dot{H} \rightarrow \frac{-\alpha}{2} H^r$	$\dot{H} \rightarrow \frac{-\alpha}{2\beta} H^{r-s}$

One can see from the table that the values of negative r must be precluded in our calculations as a result of the asymptotic behavior of the \dot{H} function. This is because for asymptotically perfect fluid behavior of our universe, the \dot{H} function

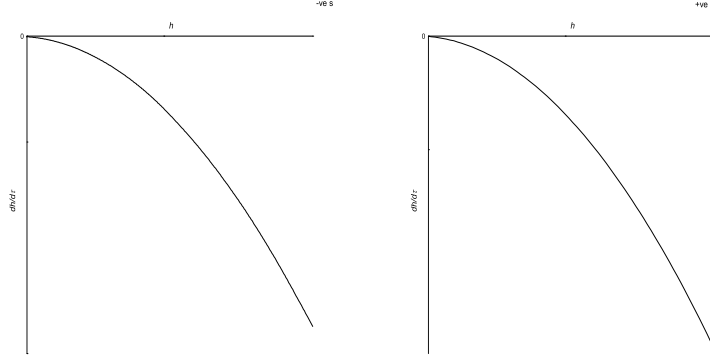


Figure 1: Fixed points phase diagram.

at large H forces r to be positive if s is negative, and forces $(r - s)$ to be positive if s is positive, which does not satisfy constraints for the pre-assumptions.

On the other hand, the asymptotic behavior of \dot{H} for the two cases of positive r can attain the perfect fluid behavior at large H if $r - s = 2$ and $\alpha/\beta = 3$ for the case of *+ve s* and if $r = 2$ and $\alpha = 3$ for the case of *-ve s*. These two cases will be studied in details.

Finally let's study the different classes of solutions due the fixed points in the model. This general class of models can be studied using the phase space method developed in [50]. The phase diagram in Fig.1 shows that there are no fixed points in the solution so that there are no different classes, here $h = \frac{H}{H_0}$ and $\tau = \frac{t}{t_0}$.

4 Case I: s is positive

This is the first case that match the dark fluid dynamics. The asymptotic behavior of the EoS at large H , early times, has the form $\dot{H} \rightarrow \frac{\alpha}{2\beta} H^{r-s}$, so that for $r - s = 2$ and $\alpha/\beta = 3$ we have the perfect fluid EoS at early times. Let's now examine the constraints that can be obtained by applying the causality condition to this model.

4.1 Causality Constraints

Spherical sound waves are lunched as a result of the overpressure due to the initial overdensity in the DM and gas. These Oscillations have a speed which is an indication to the velocity by which such perturbations are transmitted. Accordingly, it must not exceed the speed of light, which is a condition of causality. In barotropic cosmic fluid, the speed of these sound waves is defined

by the relation

$$c_s^2 = \frac{dp}{d\rho} \quad (4.1)$$

This must satisfy the condition $c_s^2 \leq 1$ that must be obeyed at all values of ρ . Accordingly, applying causality constraints, one may get an indication for the available ranges for the values of the free parameters in the EoS. Now using (2.7), we get

$$c_s^2 = -1 + \frac{[(n-m)\delta\rho^m + n]\gamma\rho^{n-1}}{(1+\delta\rho^m)^2} \leq 1 \quad (4.2)$$

Now for our case where n and m are *ve*, relation (4.2) for large ρ gives the condition

$$(n-m) \frac{\gamma}{2\delta} \rho^{n-m-1} \leq 1 \quad (4.3)$$

This is fully satisfied for

$$n \leq m+1 \quad (4.4)$$

While for small ρ , the condition will be

$$\frac{(n-m)}{2} \gamma \delta \rho^{n+m-1} + \frac{n}{2} \gamma \rho^{n-1} \leq 1 \quad (4.5)$$

This is satisfied if

$$n+m \geq 1 \quad \text{and} \quad n \geq 1 \quad (4.6)$$

Of course, this last condition is automatically satisfied if the condition in (4.4) is satisfied. Now Combining our conditions, (4.4) and (4.6), we get to the conclusion that for this case we must consider

$$1-m \leq n \leq 1+m \quad (4.7)$$

Which means

$$2(1-s) \leq r-s \leq 2 \quad (4.8)$$

In agreement with the conditions required by asymptotic behavior of the EoS at early time for this case. In fact, for the general values of ρ we also have a large dependence on γ and δ . Accordingly, we have to make a check using the obtained optimized values for the parameters.

4.2 The dynamics of the universe

In this section we study the ability of our model to produce the physical quantities that can be really measured. This, of course, depends on the compatibility of the results of our model with the astrophysical data that are indeed observed.

4.2.1 The Luminosity Distance

One of the most important observed data is that of the Hubble diagram of type SNe Ia supernovae, which is a plot of the luminosity distance D_L , that determines the flux of the source, as a function of the red shift. We will use these data to constrain the parameters in our model. Due to the asymptotic and causality constraints, we have now only two out of four parameters that are free and needed to be optimized due to observations.

To satisfy the causality and asymptotic behavior constraints, we choose $(r - s)$ as 2 while (α/β) has to be 3, so that we can attain the perfect fluid behavior of \dot{H} at large H . Let's now start with our model by using the relation (3.4) for $(r - s) = 2$ in (3.3), so that we have

$$\dot{\rho} = -3H \frac{\alpha H^{s+2}}{1 + \beta H^s} \quad (4.9)$$

which in turn gives

$$\frac{dH}{da} = -\frac{1}{2a} \frac{\alpha H^{s+1}}{1 + \beta H^s} \quad (4.10)$$

This has the solution

$$H(a) = a^{-\alpha/2\beta} \exp \left[\frac{1}{s} W \left(\frac{1}{\beta} a^{\alpha s/2\beta} e^{C \alpha s/2\beta} \right) e^{\frac{-\alpha}{2\beta} C} \right] \quad (4.11)$$

The function $W(x)$ is the Lambert W-function. To fix the constant C we consider that at some time where $t = t_0$ we have $a = a_0$ and $H = H_0$, this gives

$$C = -\ln \left(a_0 H_0^{2\beta/\alpha} \right) + \frac{2}{\alpha s} H_0^{-s} \quad (4.12)$$

So that

$$H(a) = \left(\frac{a_0}{a} \right)^{\alpha/2\beta} H_0 e^{-H_0^{-s}/\beta s} \exp \left[\frac{1}{s} W \left(\left(\frac{a_0}{a} \right)^{-\alpha s/2\beta} \frac{H_0^{-s}}{\beta} e^{H_0^{-s}/\beta} \right) \right] \quad (4.13)$$

Accordingly

$$H(z) = (z+1)^{\alpha/2\beta} H_0 e^{-H_0^{-s}/\beta s} \exp \left[\frac{1}{s} W \left((z+1)^{-\alpha/2\beta} \frac{H_0^{-s}}{\beta} e^{H_0^{-s}/\beta} \right) \right] \quad (4.14)$$

the luminosity distance D_L is then given by

$$D_L(z) = (1+z) D_p(z) \quad (4.15)$$

where $D_p(z)$ is the proper distance given by

$$D_p(z) = \int_0^z \frac{1}{H(\zeta)} d\zeta \quad (4.16)$$

Accordingly

$$D_p(z) = \frac{1}{H_0} e^{H_0^{-s}/\beta s} \int_0^z (\zeta + 1)^{-\alpha/2\beta} \exp \left[\frac{-1}{s} W \left((\zeta + 1)^{-\alpha/2\beta} \frac{H_0^{-s}}{\beta} e^{H_0^{-s}/\beta} \right) \right] d\zeta \quad (4.17)$$

The integration of the above equation can not be solved analytically. Accordingly, we will use numerical methods for solving the integral. This enables us to get an exact solution for the relation controlling the dependence of the luminosity distance on the red shift. In Fig.2 we plot the apparent magnitude, $m = M + 5 \log_{10} \left(\frac{D_L}{Mpc} \right) + 25$, of the data set from Conley et al. (2011)[51] versus their red shifts z together with the fitted results of our model. Data set are available at Supernova Cosmology Project[52]. Using $H_0 = 67.8 \text{ Km/s/Mpc}$ due to Planck+ WMAP+ BAO measurements[53], our χ^2 minimization fit results in the parameters $s = 1.435$, and $\alpha = 0.003$. These parameters are due a reduced χ^2 value of $\chi^2_{red} = 1.02$.

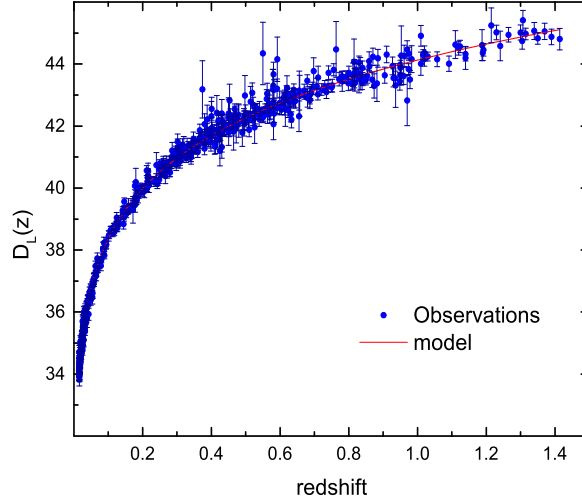


Figure 2: model versus observations for SNe Ia supernovae in case of $+ve s$

Having the full set of parameters, let's test the causality condition using these parameters. A plot of causality relation (4.2) using our parameters for this case is shown in Fig(3). We can see from the figure that the parameters satisfy the causality condition for the model throughout the whole stages of the evolution of the universe.

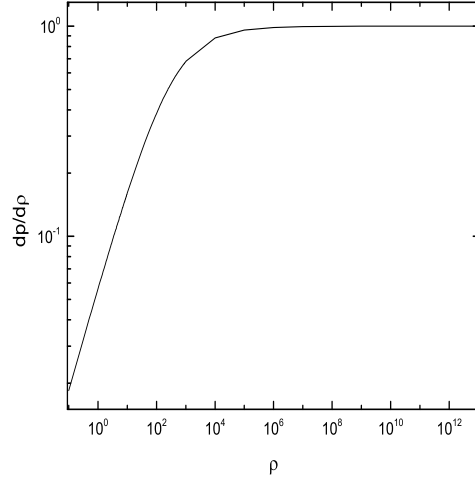


Figure 3: Causality test using our optimized free parameters for $+ve$ s

4.2.2 The Energy Density Evolution

Let us see how the energy will evolve with time due to the model in a homogeneous and isotropic universe. Relations (2.5) and (2.7) give

$$d\rho = \frac{1}{a} \frac{-3\gamma\rho^n}{1 + \delta\rho^m} da \quad (4.18)$$

To simplify our way of solution to this equation let's use our parameters. In this case we have $n = m + 1$, so that the solution of relation (4.18) will take the form

$$\rho(a) = \rho_0 \left(\frac{a}{a_0} \right)^{-3\gamma/\delta} \exp \left(\frac{W(x)}{m} - \frac{1}{m\delta\rho_0^m} \right) \quad (4.19)$$

Where

$$x = \frac{1}{\delta\rho_0^m} e^{\frac{1}{\delta\rho_0^m}} \left(\frac{a}{a_0} \right)^{3\gamma m/\delta} \quad (4.20)$$

To study the properties of relation (4.19) at early times we make use of the property

$$\exp(nW(x)) = \left(\frac{x}{W(x)} \right)^n \quad (4.21)$$

then

$$\exp \left(\frac{W(x)}{m} \right) = \left(\frac{x}{W(x)} \right)^{1/m} \quad (4.22)$$

Now, in the past, where $a \ll a_0$, so that $x \ll 1$ due to (4.20) as m is $+ve$, Taylor expansion of W -function can be approximated to $W(x) \approx x$, so that the

exponential term of the W -function converges to 1, then the energy density for that time will take the asymptotic form

$$\rho(a) \rightarrow A \left(\frac{a}{a_0} \right)^{-3} \quad (4.23)$$

as we have $\gamma/\delta = 1$ for this case, where $A = \rho_0 e^{\frac{-1}{\delta m \rho_0^m}}$ is a constant. Accordingly, the fluid of the model behaves as a matter in the earlier times.

On the other hand, for $a \gg a_0$, $x \gg 1$, the energy density function due to relation (4.22) will tend to

$$\rho(a) = (\delta x)^{-1/m} \left(\frac{x}{W} \right)^{1/m} \quad (4.24)$$

Accordingly

$$\rho(a) = \left(\frac{1}{\delta W(x)} \right)^{1/m} \quad (4.25)$$

Using the value of m we can see that this function is a very slowly decreasing function of x for $x \gg 1$, and tends to be nearly constant at $x \gg 1$, i.e. at $a \gg a_0$, a signature for a cosmological constant towards the future.

The equation of state parameter ω for this case, where $m = n - 1$, will be given, from (2.8), by

$$\omega = -1 + \frac{\gamma}{\rho^{-m} + \delta} = -1 + \frac{1}{(\rho)^{-m}/\gamma + \delta/\gamma} \quad (4.26)$$

Using our parameters for this case where $\delta/\gamma = 1$, we finally get

$$\omega = -1 + \frac{1}{(\rho)^{-m}/\gamma + 1} \quad (4.27)$$

Indicating that for earlier times, where we have large values of ρ , $\omega \rightarrow 0$, a signature for matter era, while changes to -1 gradually as the matter content of the universe diluted.

4.2.3 Deceleration Parameter and the Universe Acceleration

The acceleration of the universe is related to a dimensionless parameter known as the deceleration parameter q defined as

$$q = -\frac{\ddot{a}}{a} \frac{1}{H^2} \quad (4.28)$$

Since you have to have $\frac{\ddot{a}}{a} > 0$ to get an accelerating universe, the deceleration parameter will then measure if the expansion of the universe is accelerating or decelerating. If $q < 0$, the expansion is accelerating, while if $q > 0$ it is decelerating. Now as

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} \quad (4.29)$$

Then

$$q = -1 - \frac{\dot{H}}{H^2} \quad (4.30)$$

Now using (3.4), one gets the deceleration parameter for our model to be

$$q = -1 + \frac{\alpha H^{r-2}}{2(1 + \beta H^s)} \quad (4.31)$$

Using $r = s + 2$, relation (4.31) gives

$$q = -1 + \frac{\alpha}{2(H^{-s} + \beta)} \quad (4.32)$$

Using the values of our parameters with $H_0 = 67.8$, the present day deceleration parameter due to this model case has the value

$$q_0 = -0.553 \quad (4.33)$$

Relation (4.32) also shows that deceleration parameter evolves towards more negative values as the Hubble parameter decreases with time.

5 Case II: s is negative

For this case $\dot{H} \rightarrow \frac{-\alpha}{2}H^r$ asymptotically at large H , which means that we can attain the perfect fluid EoS at early time if $r = 2$ and $\alpha = 3$. However, a first step again is the examination of the causality constraints for this case of the model.

5.1 Causality Constraints

In this case m is $-ve$, so that relation (4.2) at large values of ρ tends to

$$\frac{1}{2}n\gamma\rho^{n-1} \leq 1 \quad (5.1)$$

Which is satisfied if

$$0 < n \leq 1 \quad (5.2)$$

while for small ρ we have

$$\left(n - \frac{m}{2}\right) \frac{\gamma}{\delta} \rho^{n-m-1} \leq 1 \quad (5.3)$$

As m is $-ve$, this is always satisfied under the condition

$$n \geq 1 \quad (5.4)$$

Combining (5.2) and (5.4) we get to the conclusion that in this case we must consider

$$n = 1 \quad (5.5)$$

which means

$$r = 2 \quad (5.6)$$

in full agreement with the asymptotic behavior of the EoS at early time for this case.

5.2 The dynamics of the universe

We'll now study the physical quantities representing the dynamics of our universe due to this case of the model and comparing our results with observations.

5.2.1 The Luminosity Distance

We could now constrain two parameters out of four through asymptotic behavior of the model and the causality condition, where r has to be 2 while α has to be 3. Let's then calculate the luminosity distance function $D_L(z)$ and use the observed data of this function to constrain the remaining two parameters of the model. We start by using relation (3.4) for $r = 2$ in (3.3), so that we have

$$\dot{\rho} = -3H \frac{\alpha H^2}{1 + \beta H^s} \quad (5.7)$$

which in turn gives

$$\frac{dH}{da} = -\frac{1}{a} \frac{\alpha H}{1 + \beta H^s} \quad (5.8)$$

This has the solution

$$H(a) = a^{-\alpha/2} \exp \left[-\frac{1}{s} W \left(\beta a^{-\alpha s/2} e^{-C\alpha s/2} \right) - \frac{C\alpha}{2} \right] \quad (5.9)$$

And let's assume that at some time $t = t_0$ we have $a = a_0$ and $H = H_0$. This gives

$$C = -\ln \left(a_0 H_0^{2/\alpha} \right) - \frac{2}{\alpha s} \beta H_0^s \quad (5.10)$$

So that

$$H(a) = \left(\frac{a_0}{a} \right)^{\alpha/2} H_0 \exp \left(\frac{\beta H_0^s}{s} \right) \exp \left[-\frac{1}{s} W \left(\beta \left(\frac{a_0}{a} \right)^{\alpha s/2} H_0^s e^{\beta H_0^s} \right) \right] \quad (5.11)$$

In turn, we can write $H(z)$ as

$$H(z) = (z+1)^{\alpha/2} H_0 \exp \left(\frac{\beta H_0^s}{s} \right) \exp \left[-\frac{1}{s} W \left(\beta (z+1)^{\alpha s/2} H_0^s e^{\beta H_0^s} \right) \right] \quad (5.12)$$

The luminosity distance D_L , Eqs (4.15), is then calculated using the proper distance which due to (4.16) will be given by

$$D_p(z) = \frac{1}{H_0} \exp \left(-\frac{\beta H_0^s}{s} \right) \int_0^z (\zeta+1)^{-\alpha/2} \exp \left[\frac{1}{s} W \left(\beta (\zeta+1)^{\alpha s/2} H_0^s e^{\beta H_0^s} \right) \right] d\zeta \quad (5.13)$$

Again, the integration of relation (5.13) can not be solved analytically so that we use numerical methods. The apparent magnitude of Conley et al. (2011)[51] data set is plotted in Fig.4 together with our results. Our χ^2 minimization fit results in the parameters $s = -2.1$ and $\beta = 1.64 \times 10^4$. These parameters are due a χ^2 value of $\chi^2_{red} = 1.23$.

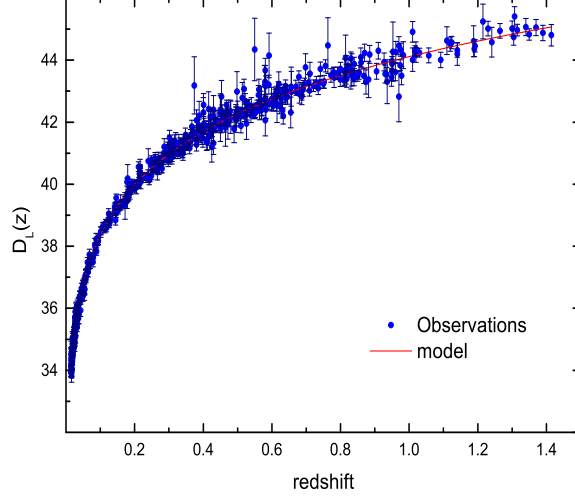


Figure 4: model versus observations for SNe Ia supernovae for $-ve s$

A last step for this case now is to test the causality condition using these parameters. A plot of causality relation (4.2) using our parameters is shown in Fig(5). The figure shows that the causality is satisfied for all values of ρ , i.e., it is ensured by the model in this case through the universe evolution.

5.2.2 The Energy Density Evolution

Let's solve relations (4.18) for $n = 1$, the value of the parameter n in this case. This gives

$$\rho(a) = \rho_0 \left(\frac{a}{a_0} \right)^{-3\gamma} \exp \left(\frac{-W(x)}{m} + \frac{\delta\rho_0^m}{m} \right) \quad (5.14)$$

Where

$$x = \delta\rho_0^m e^{\delta\rho_0^m} \left(\frac{a}{a_0} \right)^{-3\gamma m} \quad (5.15)$$

Studying the properties of relation (5.14) we see that due to relation (4.21) we have

$$\exp \left(\frac{-W(x)}{m} \right) = \left(\frac{x}{W(x)} \right)^{-1/m} \quad (5.16)$$

so that in the past where $a \ll a_0$, so that $x \ll 1$ due to (5.15) since m is $-ve$, we can consider the W -function as approximated to $W(x) \approx x$ and its exponential term converges to 1, the energy density for that time will then has the asymptotic form

$$\rho(a) \Rightarrow A \left(\frac{a}{a_0} \right)^{-3} \quad (5.17)$$

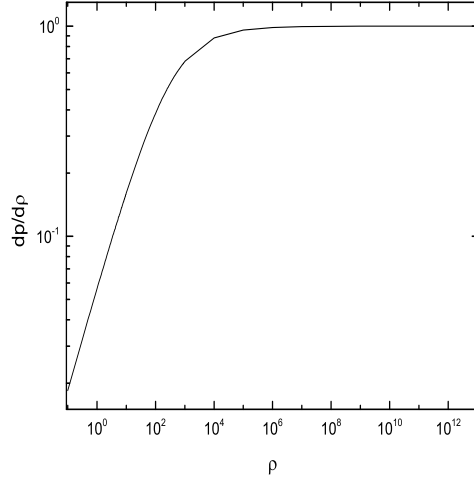


Figure 5: Causality test using our optimization parameters for $-ve s$

Where we have $\gamma = 1$ for this case and $A = \rho_0 e^{\delta \rho_0^m / m}$. This shows that, the fluid of our model behaves as a matter in the earlier times.

On the other hand, to study the fluid for $a \gg a_0$, $x \gg 1$, we make use of relation (5.15) together with the property of relation (5.16) in the energy density function, relation (5.14), so that the energy density function takes the form

$$\rho(a) = \left(\frac{x}{\delta}\right)^{1/m} \left(\frac{x}{W}\right)^{-1/m} \quad (5.18)$$

Accordingly

$$\rho(a) \Rightarrow \left(\frac{\delta}{W(x)}\right)^{-1/m} \quad (5.19)$$

Using the value of the parameter m we can see that this function is again a very slowly decreasing function of x for $x \gg 1$ and tends to be nearly constant at $x \gg 1$, i.e., at $a \gg a_0$, which is again a signature for the cosmological constant towards the future.

Finally, the equation of state parameter ω for this case will be given from (2.8) by

$$\omega = -1 + \frac{1}{1 + \delta \rho^m} \quad (5.20)$$

Indicating that for large values of ρ , where we are deep in the matter era, $\omega = 0$ as m is $-ve$, while changes to -1 gradually as the matter content of the universe diluted.

5.2.3 Deceleration Parameter and the Universe Acceleration

Using $r = 2$ and $\alpha = 3$ in relation (4.31) we get

$$q = -1 + \frac{3}{2(1 + \beta H^s)} \quad (5.21)$$

and using the values of s and β together with $H_0 = 67.8$, this relation gives a present day deceleration parameter of

$$q_0 = -0.551 \quad (5.22)$$

Relation (5.21) also shows that as s is negative, the deceleration parameter evolves towards more negative values as the Hubble parameter decreases to the future.

6 Conclusions

A barotropic fluid model is considered which initially can be described as a correction to the dark energy regime by a power law. The power law enables interpolation between the DM and DE equations of state, and so guarantees a smooth phase transition from matter era to DE era. Accordingly, our model is a dark fluid model provides a unification for the two dark sectors in one EoS that has the advantage of the general asymptotic solution. This means that it can describe dark matter at early times while describing dark energy at late times, while adopts a general EoS for DE which enables the cosmological constant as a special case.

Studying the asymptotic behavior of the equation of state constrains the parameters of the equation and restricts the model. It also clarifies the cases that can be physically acceptable as dark fluid which are found to be two cases. On the other hand, studying the phase diagram of the model for each of these two cases shows that there are no fixed points for both, with the result that there is only one class of solution for each of them.

Causality condition further constrains the parameters of the model, so that at the end, only two out of four parameters are free. These two parameters are constrained using the Hubble diagram of SNe Ia supernovae, where we present an exact solution for the red shift dependence of the luminosity distance in each model, or model case, avoiding the approximate formulas that are accurate only at small red shifts. The χ^2 -fitting of the two models shows that they both fit the data well. For the model with $+ve - s$ we got a $\chi^2 = 1.02$, and for the model with $-ve - s$ we got a $\chi^2 = 1.23$. However, it is clear that observational data prefer the $+ve - s$ model than the $-ve - s$ one. More analysis such as density perturbation analysis may differentiate them and may favor one of the models than the other.

We also studied the energy density evolution which ensures the smooth transition between the two dark sectors. Furthermore, we examined the evolution of the EoS parameter ω that again ensures its two extremes of 0 and -1 for

early and late times. We also Calculated the deceleration parameter q and examined its evolution. Calculations resulted in a negative q for both model cases with the present day values of -0.553 and -0.551 due to the two models, while assures evolution towards more negativity to the future as the Hubble parameter decreases, a clear evidence for accelerated expansion in agreement with observations.

Due to the above mentioned results, our model can be considered as a dark fluid model unifying the two dark sectors of the universe and succeeded in describing the evolution of the universe.

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References

- [1] Riess A.G., et. al., [Supernova Search Team Collaboration], *Astrophys. J.*, **607**, 665, 2004.
- [2] Knop R.A., et. al. [Supernova cosmology Project Collaboration], *Astrophys. J.*, **598**, 102, 2003.
- [3] Riess A.G., et. al., [Supernova Search Team Collaboration], *Astron. J.*, **116**, 1009, 1998.
- [4] Tegmark M., et. al., [SDSS Collaboration], *Phys. Rev.* **69**, 103501, 2004; Tegmark M., et. al., [SDSS Collaboration], *Astrophys. J.*, **606**, 702, 2004.
- [5] Bennett, C.L., et. al., *Astrophys. J. Suppl.*, **148**, 1, 2003.
- [6] de Bernardies, P., et. al., *Nature*, **404**, 955, 2000.
- [7] Spergel, D.N., et. al., *Astrophys. J. Suppl.*, **148**, 175, 2003.
- [8] Spergel D. N., et al., [WMAP Collaboration], *Astrophys. J. Suppl. Ser.* **170**, 377, 2007.
- [9] Eisenstein D. J., et al., *Astrophys. J.* **148**, 175, 2003.
- [10] Stomper R., et. al., *Astrophys. J.*, **561**, L7, 2001.
- [11] Allen S.W., et.al., *Mon. Not. Roy. Astron. Soc.* **353**, 457, 2004.
- [12] Riess A.G., et al., *Astrophys. J.* **659**, 98, 2007.
- [13] Perlmutter S. J., et al., *Nature*, **391**, 51, 1998.
- [14] Tonry S. L., et al., *Astrophys. J.* **594**, 1, 2003.

- [15] Yue-Yao Xu, Zhang X., astro-ph/1607.06262, 2016.
- [16] Copeland E.J., Sami M., and Tsujikawa S., int. J. Mod. Phys., **D15**, 1753, 2006.
- [17] Peebles P.J.E. and Ratra B., Astrophys. J., **325**, L17-L20, 1988.
- [18] Wienberg S., Rev. of mod. phys., **61**, 1, 1989.
- [19] Carroll S.M., Press W.H., and Turner E.L., Annu. Rev. Astron. Astrophys., **30**, 499, 1992.
- [20] Sahni V., and Starobinsky A.A., int. J. Mod. Phys., **D9**, 373, 2000.
- [21] Padmanabhan T., Phys. Rept., **380**, 235, 2003.
- [22] Carroll S.M., LivingRev.Rel., **4**, 1, 2001.
- [23] Wehus I.K., and Ravndal F., J of Phys.: Conference Series **66**, 012024, 2007.
- [24] Martinez C., Troncoso R., and Zanelli J., Phys. Rev. **D70**, 084035, 2004.
- [25] Gazeau J.P., Renaud J., and Takook M.V., Class. Quantum Grav. **17**, 6, 1415, 2000.
- [26] Allen B., and Folacci A., Phys. Rev. **D35**, 3771, 1987.
- [27] Cordero R., Gonzalez E.L., and Queijeiro A., arXiv:1608.06540, 2016.
- [28] Almeida C.R., Fabris J.C., Sbis F., and Tavakoli Y., arXiv:1604.00624, 2016.
- [29] Guendelman E., Nissimov E., and Pacheva S., Europ. Phys. J., **C76**, 90, 2016.
- [30] Matsumoto J., Nojiri S., Phys.Lett., **B687**, 236, 2010.
- [31] Baffou E.H., Salako I.G., and Houndjo M.J.S., arXiv:1606.05265, 2016.
- [32] Sharov G.S., JCAP **06**, 023, 2016.
- [33] Dinda B.R., Kumar S., Sen A.A., Phys. Rev. D, **90**, 083515, 2014.i
- [34] Ludwick K.J., Phys. Rev. D, **92**, 063019, 2015.
- [35] Basilakos S., and Sola J., Mon. Not. Roy. Astron. Soc., **437**, no. 4, 3331, 2014.
- [36] Alcaniz J.S., Braz. J., Phys. J. Suppl., **36**, 1109, 2006.
- [37] Ellis J.R., Phil. Trans. Roy. Soc. Lond., **361**, 2607, 2003.

- [38] Alam S., et. al., Mon. Not. Roy. R. Astron. Soc., **000**, 1, 2016, arXiv:astro-ph/1607.03155, 2016.
- [39] Rivera A.B., and Farieta J.G., arXiv:astro-ph/1605.1098, 2016.
- [40] Bamba K., and Odintsov S.D., Symmetry, **7**, 220, 2015.
- [41] Modak B., Sarkar K., and Sanyal A.K., Astrophysics and Space Science, **353**, Issue 2, 707, 2014.
- [42] Kamenshchik A.Y., Moschellai U., and Pasquier V.M., Phys. Lett. **B511**, issues 2-4, 265, 2001.
- [43] Bento M.C., U., Bertolami O., and Sen A.A., Phys. Rev. **D66**, 043507, 2002.
- [44] Benaoum H.B., arXiv:hep-th/0205140v1, 2002.
- [45] Bilić N., Tupper G.B., and Viollier R.D., JCAP **0510**, 003, 2005.
- [46] Nojiri S., Odintsov S.D., Phys. Rev. **D70**, 103522, 2004.
- [47] Štefančić H., Phys.Rev. **D71**, 084024, 2005.
- [48] Štefančić H., arXiv:astro-ph/0609780.
- [49] Nojiri S., Odintsov S.D., Tsujikawa S., Phys.Rev. **D71**, 063004, 2005.
- [50] Awad A. Phys.Rev. **D87**, no.10, 103001, 2013.
- [51] Conley A., et al., ApJS, **192**, 1, 2011.
- [52] Supernova Cosmology Project: <http://supernova.lbl.gov/Union/>.
- [53] Plank collaboration, A&A, 2015; arXiv: astro-ph/1502.01582, 2015.